RTH 104-93

STATISTICAL CONSIDERATIONS

1. Scope

1.1 The purpose of this recommended practice is to outline some general statistical concepts which may be applied to small sample sizes typical of rock test data. It is not the intent to deal with accuracy or precision considerations, but rather to assess the results from the assumed point that the test has been conducted as specified in the respective test methods.

2. Variation and Sample Size

2.1 Most physical tests involve tabulation of a series of readings, with computation of an average said to be representative of the whole. The question arises as to how representative this average is as the measure of the characteristic under test. Three important factors introduce uncertainties in the result:

(a) Instrument and procedural errors.

(b) Variations in the sample being tested.

(c) Variations between the sample and the other samples that might have been drawn from the same source.

If a number of identical specimens were available for tests, or if the tests were nondestructive and could be repeated a number of times on the same specimens, determination of the procedural and instrument errors would be comparatively simple, because in such a test the sample variation would be zero. Periodic tests on this specimen or group of specimens could be used to check the performance of the test procedure and equipment. However, as most of the tests used in determining the mechanical properties of rock are destructive, the instrument and sample variations cannot be separated. Nevertheless, we may apply some elementary statistical concepts and still have confidence in the test results. Of course, the more test data available the more reliable the results. Due to the expense of testing occasionally the shortage of test specimens, rock test data almost always require treatment as groups of small samples. As a general rule at least 10 tests are recommended for any one condition of each individual test with an absolute minimum of 5.

3. Measures of Central Tendency and Deviation

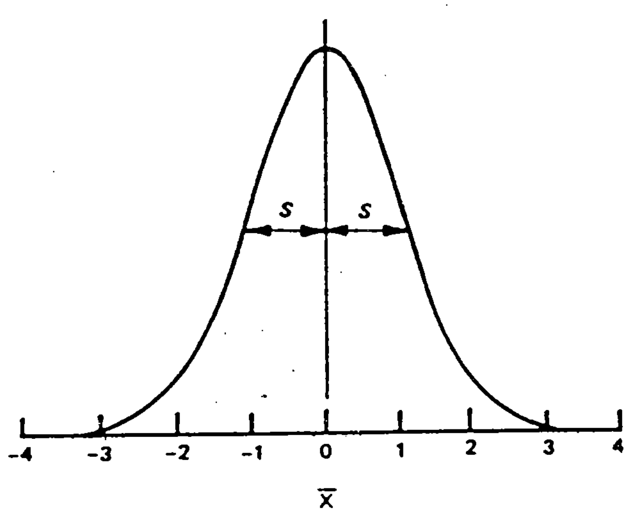
3.1 The most commonly used measure of the central tendency of a sample of n specimens is the arithmetic mean or average, $\bar{x}$. The standard deviation, s, is a measure of the sample variability. It is calculated as follows:

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

where x_i's are the individual observations. Most calculators have a built-in program that calculates standard deviation. Some have two programs, one that calculates s based on $n-1$ degrees of freedom and one that calculates s based on n degrees of freedom. The one that uses $n-1$ degrees of freedom is correct for most applications.

4. The Normal Distribution

4.1 A series of tests for any one property will, of course, have some variation in the individual determinations. The distribution of these bits of data quite often follows a pattern of normal distributions, i.e., a large portion of the data bits will be closely grouped to the mean on either side with progressively fewer bits distributed farther from the mean. The normal distribution is usually portrayed graphically as shown below.



Methods are available to check the normality of the distribution and to deal with those which are not normal (skewed).[7.1-7.3] Use of many statistical techniques requires the assumption that the data be normally distributed, but mild cases of skewness do not normally cause severe errors and can be ignored. A more important assumption is that the samples be taken at random from the population, as discussed in paragraph 5.

5. Sampling

5.1 It is extremely important that samples from a parent population be taken at random. Probably the most frequent cause of incorrect conclusions being drawn about a population from a sample is non-random sampling. Sometimes it is impossible to draw random samples because of physical or cost limitations on a project. When this happens, interpretations of results should be made with caution.

5.2 A study of sampling distributions of statistics for small samples ($n < 30$) is called small sampling theory. Statistical tables are available for use with the proper relationship to develop confidence in test data.[7.1] For small samples the confidence limits for a mean are given by:

$$\bar{x} \pm t_c \frac{s}{\sqrt{n - 1}}$$

$\bar{x}$, n , and s are determined as given in paragraph 3.1. t_c is from and depends on the level of confidence desired and the sample size. Values of t_c for 90, 95, and 99 percent confidence limits are given below for n from 3 to 12.

n	DF*	t_c, % 90	95	99
3	2	2.92	4.30	9.92
4	3	2.35	3.18	5.84
5	4	2.13	2.78	4.60
6	5	2.02	2.57	4.03
7	6	1.94	2.45	3.71
8	7	1.90	2.36	3.50
9	8	1.86	2.31	3.36
10	9	1.83	2.26	3.25
11	10	1.81	2.23	3.17
12	11	1.80	2.20	3.11

* Degrees of freedom

Using this calculation and given an estimate of the standard deviation of a population, one can estimate the number of samples needed to get a desired level of confidence on the mean. See paragraph 7.2 for an example.

6. <u>Dealing with Outliers</u>

6.1 An outlying observation or "outlier" is one that appears to deviate markedly from other members of the sample in which it occurs. Outliers may be merely unusual examples of the population extremes or they may actually be samples taken accidentally from another population. In the former case, the observations should not be deleted from the sample, whereas in the latter case they should be deleted. It is often difficult to know which applies. When some assignable cause is known, for example when one rock specimen is allowed to dry out prior to strength testing and all others are wet, then the dry specimen is reasonably considered to belong to a different population and should be discarded. In the absence of an assignable cause, outliers should not be discarded without first examining the observation in light of an objective procedure. This is particularly critical with small samples. For example, it is quite common when samples of 3 are taken that two observations fall quite close together and the third to be somewhat away. Without substantial experience or an objective method, intuition often misleads one into discarding the outlier. Many test methods provide guidance on handling outliers. ASTM E 178[7.5] provides a general technique for handling outliers.

7. <u>Examples</u>

7.1 For the purposes of illustrating a confidence limit calculation, assume there is a normal distribution of compressive strength data[7.4] for a particular rock type yielding the following individual specimen strengths, taken at random: 18,000; 18,700; 19,200; 19,600; 20,000; 20,100; 20,500; 20,800; 21,100; and 22,000 psi. Find the 95 percent confidence limits for the mean strength.

By computation:

$$n = 10$$

$$\overline{x} = 20{,}000 \text{ psi}$$

$$s = 1183$$

$$95\% \text{ confidence limits} = \overline{x} \pm t_{95}\ (s/\sqrt{n-1});\ \text{thus}$$

$$\overline{x} \pm 2.26(1183/\sqrt{10-1}) =$$

$$20{,}000 \pm 2.26\ (420) = 20{,}000 \pm 891 \text{ psi}$$

Thus, we can be 95 percent confident that the true mean lies between 19,109 and 20,891.

7.2 As an example of the use of confidence limit calculation to estimate sample sizes, consider the population described in paragraph 7.1. Suppose the objective of a sample was to estimate the mean strength of that population with a confidence limit on that mean of ±1,000 psi. Then confidence limit could be calculated for a range of sample sizes, as follows:

Sample Size	95% Confidence Interval
4	1636 psi
5	1238 psi
6	1024 psi
7	891 psi

By inspection, a sample size of about 6 should be suitable to achieve the desired confidence limit.

8. References

8.1 Spiegel, M. R., Theory and Problems of Statistics, Schaun's Outline Series, McGraw-Hill Company, New York, 1961.

8.2 Volk, William, Applied Statistics for Engineers, McGraw-Hill Company, New York, 1958.

8.3 Obert, Leonard and Duvall, W. I., Rock Mechanics and the Design of Structures in Rock, John Wiley and Sons, Inc., New York, 1967.

8.4 "Engineering Geology; Special Issue, Uniaxial Testing in Rock Mechanics Laboratories," Elsevier Publishing Company, Amsterdam, Vol 4, No. 3, July 1970.

8.5 ASTM E 178. "Standard Practice for Dealing with Outlying Observations," Annual Book of ASTM Standards, Vol 14.02, ASTM, Philadelphia, PA.